

Proving that a Quadrilateral is a Parallelogram

son Objective

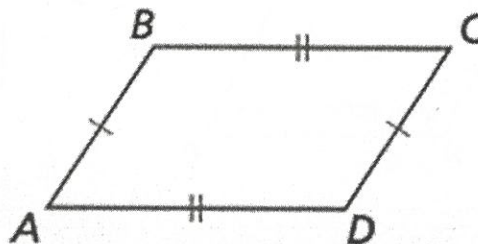
INBAT identify and verify ~~the~~ 11-grams; show that a quadrilateral is a 11-gram in the coordinate plane.

You can only prove that a quadrilateral is a parallelogram using its definition (below):
a quad. w/ 2 pairs of opp. $\parallel$ sides.

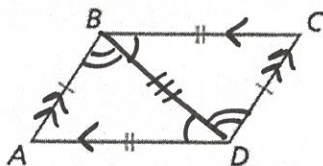
However, there are theorems that allow us to use other properties of quadrilaterals to conclude that it is a parallelogram. They are as follows:

Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



PROOF: **Given:** $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
Prove: ABCD is a parallelogram



Statements

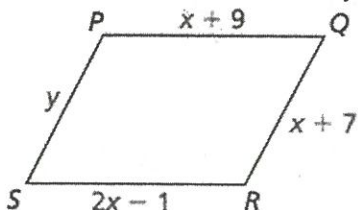
Reasons

- | | |
|--|---|
| ① $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ | ① given |
| ② Draw $\overline{BD}$ | ② Thru any 2 points there is one line |
| ③ $\overline{BD} \cong \overline{DB}$ | ③ reflex. prop. $\cong$ |
| ④ $\triangle ABD \cong \triangle CDB$ | ④ SSS $\cong$ (1, 1, 3) |
| ⑤ $\angle ABD \cong \angle CDB$
$\angle ADB \cong \angle CBD$ | ⑤ CPCTC |
| ⑥ $\overline{BC} \parallel \overline{AD}$
$\overline{AB} \parallel \overline{CD}$ | ⑥ if alt. int. $\angle s \cong \rightarrow \parallel$ |

Example:

⑦ ABCD is a 11-gram ⑦ def. of a 11-gram

1. For what values of x and y is quadrilateral PQRS a parallelogram?



$$x+9 = 2x-1$$

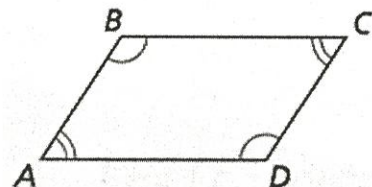
$$\boxed{x=10}$$

$$y = x+7$$

$$\boxed{y=17}$$

Parallelogram Opposite Angles Converse

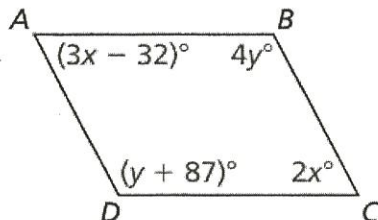
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



PROOF: end of class (if time permits). Will be on Edline tonight if you are so curious to look it up ☺

Example:

2. For what values of x and y is quadrilateral ABCD a parallelogram?



$$3x - 32 = 2x$$

$$\boxed{x = 32}$$

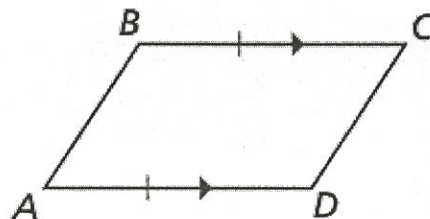
$$y + 87 = 4y$$

$$3y = 87$$

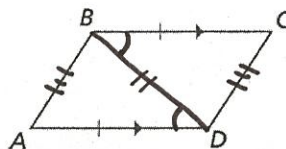
$$\boxed{y = 29}$$

Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



PROOF: **Given:** $\overline{BC} \cong \overline{AD}$ and $\overline{BC} \parallel \overline{AD}$
Prove: ABCD is a parallelogram



Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements

Reasons

① $\overline{BC} \cong \overline{AD}$ and $\overline{BC} \parallel \overline{AD}$

① given

② Draw $\overline{BD}$

② Thru any 2 points there is one line

③ $\overline{BD} \cong \overline{DB}$

③ reflex. prop. $\cong$

④ $\angle CBD \cong \angle ADB$

④ if $\parallel \rightarrow$ alt. int. $\angle$ s $\cong$

⑤ $\triangle ADB \cong \triangle CBD$

⑤ SAS $\cong$ (1, 4, 3)

⑥ $\overline{AB} \cong \overline{CD}$

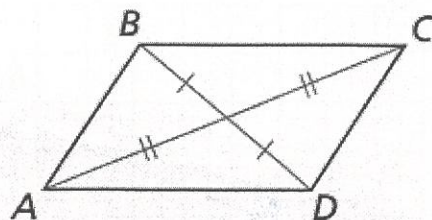
⑥ CPCTC

⑦ ABCD is a $\parallel$ -gram

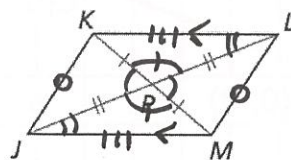
⑦ If opp. sides $\cong \rightarrow \parallel$ -gram

Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other,
then the quadrilateral is a parallelogram.



PROOF: **Given:** Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other
Prove: JKLM is a parallelogram



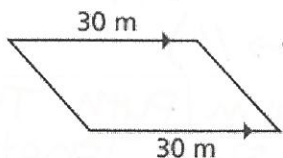
Remember, you can use any past theorems you proved to be true as reasons!!!!

Statements	Reasons
① Diags $\overline{JL}$ and $\overline{KM}$ bisect each other	① given
② $\overline{JP} \cong \overline{LP}$ and $\overline{KP} \cong \overline{MP}$	② def. of a seg. bisector
③ $\angle KPL \cong \angle MPJ$	③ vert. $\angle$ s $\cong$
④ $\triangle KPL \cong \triangle MPJ$	④ SAS $\cong$ (2, 3, 2)
⑤ $\overline{KL} \cong \overline{MJ}$	⑤ CPTC
$\angle KLP \cong \angle MJP$	
⑥ $\overline{KL} \parallel \overline{MJ}$	⑥ if alt. int. $\angle$ s $\cong \rightarrow \parallel$
⑦ JKLM is a $\parallel$ -gram	⑦ if same pair of opp. sides $\cong$ and $\parallel \rightarrow \parallel$ -gram

Example:

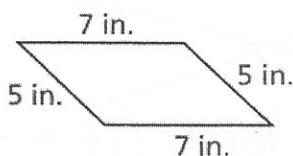
3. State the theorem you can use to show that the quadrilateral is a parallelogram.

a.



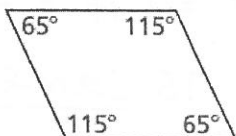
if same pair of opp. sides $\cong$ and $\parallel \rightarrow \parallel$ -gram

b.



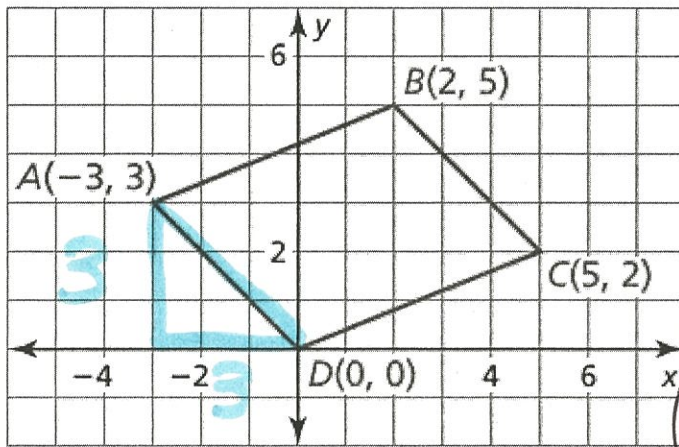
if opp. sides $\cong \rightarrow \parallel$ -gram

c.



if opp. $\angle$ s $\cong \rightarrow \parallel$ -gram

4. Show that quadrilateral ABCD is a parallelogram.



$$\text{slope } \overline{AD} = -\frac{3}{2}$$

$$\text{slope } \overline{BC} = -\frac{3}{2}$$

$$\text{slope } \overline{AB} = \frac{2}{5}$$

$$\text{slope } \overline{CD} = \frac{2}{5}$$

Since the slopes of the opp. sides of the quadrilateral are $=$, then those opp. sides are $\parallel$. So, ABCD is a $\parallel$ -gram.

Ways to Prove a Quadrilateral is a Parallelogram		
Theorem or Definition	Diagram	Method for Coordinate Plane
Show that both pairs of opposite sides are parallel.		Find slopes of opp. sides (if $= \rightarrow \parallel$)
Show that both pairs of opposite sides are congruent.		use dist. form / Pyth Thm to find opp. side lengths (if $= \rightarrow \cong$)
Show that both pairs of opposite angles are congruent.		
Show that one pair of opposite sides are congruent and parallel.		use slopes for $\parallel$ (if $= \rightarrow \parallel$) use Pyth Thm for $\cong$ (if $= \rightarrow \cong$)
Show that the diagonals bisect each other.		show mdpt is <u>same</u> for both diags.